CENTRE FOR ADVANCEMENT OF STANDARDS IN EXAMINATIONS

(GEMS – ASIAN SCHOOLS)

COMMON REHEARSAL EXAMINATIONS–JANUARY 2010

(ALL INDIA SENIOR SCHOOL CERTIFICATE EXAMINATION) **FOR GRADE XII**

MATHEMATICS

TIME: 3 Hrs. **Max. Marks:100**

General Instructions:

1. **All questions are compulsory.**
2. **The question paper consists of 29 questions divided into sections A,B and C. Section A comprises of ten questions of 1 mark each. Section B comprises of 12 questions of 4 marks each. Section C comprises of 7 questions of 6 mark each.**
3. **All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.**
4. **Use of calculators is not permitted. However you may ask for mathematical tables.**
5. **There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.**

SECTION A

1. Find the principal value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.
2. Let A = $\{1,2,3\}$ B = $\{4,5\}$ C = $\{5,6\}$. Let f:A→ B, g:B→ C be defined by f(1)=4, f(2)=5, f(3)=4, g(4)=5, g(5)=6. Find gof : A→ C.
3. A is a non-singular matrix of order 3 and $|A|$ = –4. Find $|adjA|$.
4. If A = diag[2,-5,9], B = diag[-3, 7, 14] and C = diag[4,-6,3] find 2A + B – 5C.
5. Evaluate $\int\sqrt{1+\sin x}\,dx$.
6. Find the slope of the tangent to the curve y = $(\sin 2x + \cot x + 2)^2$ at x = $\frac{\pi}{2}$.
7. If $\begin{vmatrix} 2 & x \\ -1 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix}$, find x.
8. Find $|\vec{a}-\vec{b}|$ if vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|$ = 2, $|\vec{b}|$ = 3 and $\vec{a}\cdot\vec{b}$ = 4.
9. Find the area of the parallelogram whose diagonals are along the vectors $2\hat{i}$ and $3\hat{k}$.
10. Find the intercepts cuts by the plane 3x–2y+4z–12 = 0 on axes.

SECTION B

11. Prove that $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$

OR

Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

12. Using properties of determinants prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

13. Let $f:Z \rightarrow Z : f(n)=3n$ and Let $g:Z \rightarrow Z$ defined by

$$g(n) = \begin{cases} \frac{n}{3} & \text{if } n \text{ is a multiple of } 3 \\ 0 & \text{if } n \text{ is not a multiple of } 3 \end{cases}$$

Show that gof = Iz and fog $\neq$ Iz.

14. Find the equation of the normal lines to the curve $y = 4x^3-3x+5$ which are parallel to the line $9y+x+3 = 0$.

15. Find the derivative of $5^{\log\sin x}+(\sin x)^x$ with respect to x.

OR

Verify Rolle's theorem for the function $f(x) = \sin x + \cos x$, $x \in [0, \Pi/2]$

16. Find all the points of discontinuity of f defined by $f(x) = |x|-|x+1|$

17. Evaluate $\int 5^{5^{5^x}} . 5^{5^x} . 5^x \, dx$ **OR** Evaluate $\int \frac{1}{\sin(x-p)\cos(x-q)} dx$.

18. Find the value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and

$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

19. If $\hat{a}$ and $\hat{b}$ are two unit vectors and θ is the angle between them,

then show that $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$.

20. Form the differential equation representing the family of

ellipses having foci on x axis and centre at origin.

21. Solve the differential equation $\frac{dy}{dx} = x^5\tan^{-1}(x^3)$.

OR

Solve the differential equation $(1+e^{x/y})dx + e^{x/y}(1-\frac{x}{y})dy = 0$.

22. An anti aircraft gun can take a maximum of three shots at an

enemy plane moving away from it. The probability of hitting the

plane at first, second and third shot are $\frac{2}{3}$, $\frac{2}{5}$, and $\frac{3}{8}$ respectively.

What is the probability that plane is hit?

SECTION C

23. Find the matrix A such that $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} A \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

OR

Using the elementary transformation, find the inverse of the

matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

24. Evaluate the integral $\int_{-1}^{2} [\,|x+1| + |x| + |x-1|\,]\,dx$.

25. Calculate the area of the region enclosed between the circles

$x^2+y^2 = 1$ and $(x - \frac{1}{2})^2 + y^2 = 1$

OR

Find the area of the region

$\{(x,y) : x^2 + y^2 \le 2ax, y^2 \ge ax, \quad x \ge 0, \quad y \ge 0\}$

26. Find the distance of the point (2,2,-1) from the plane $x+2y-z = 1$

measured parallel to the line $\frac{x+1}{2} = \frac{y+1}{2} = \frac{z}{3}$.

27. Two cards are drawn simultaneously from a well-shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

28. A medical company has factories at two places A and B. From these places, supply is made to each of its three agencies at P,Q and R. The monthly requirement of the agencies are respectively 40, 40 and 50 packets of the medicines, while the production capacity of factories A and B are 60 and 70 packets respectively. The transportation cost per packet from the factories to the agencies are given below.

Transportation cost per packet		
To \ from	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each factory be transported to each agency, so that the cost of the transportation is minimum? Also find the minimum cost?

29. Prove that, the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone, is half of that of the cone.